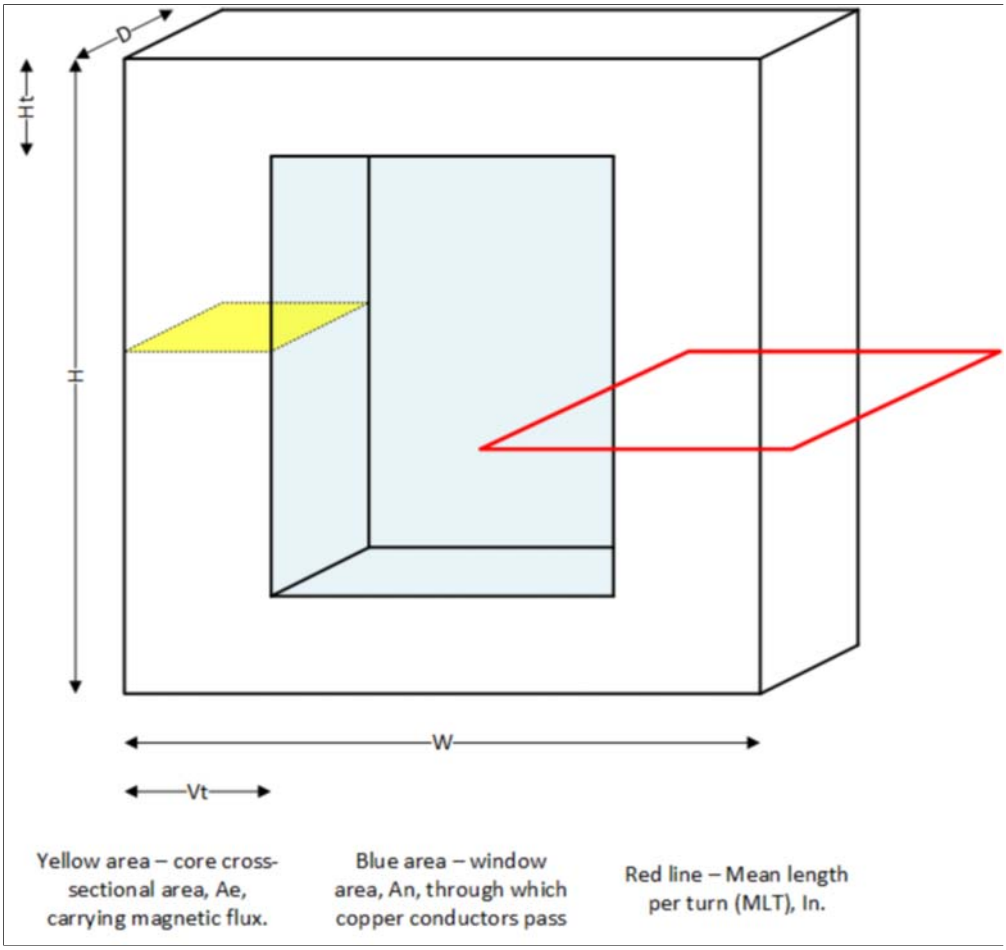
Optimum UU Core dimentions maximizing Kg based on outer length and width dimensions

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Core diagram

Figure 1: Diagram of UU core showing the main core dimensions, the window area, the core cross-sectional area and the mean length per turn.



Assume that Vt = Ht to simplify the analysis.

(%i19) kill(all)$

The internal width of the core

(%i1) EQ1:Wi=W−Vt·2;

(EQ1) $Wi = W - 2\ Vt$

The internal height of the core

(%i2) EQ2:Hi=H−Vt·2;

(EQ2) $Hi = H - 2\ Vt$

The core cross-sectional area

(%i3) EQ3:Ae=D·(Vt+Vt)/2;

(EQ3) $Ae = D\ Vt$

The core window area

(%i4) EQ4:An=(H−(2·Vt))·(W−(2·Vt));

(EQ4) $An = (H - 2\ Vt)\ (W - 2\ Vt)$

The area product

(%i5) EQ5:Ap=Ae·An;

(EQ5) $Ap = Ae\ An$

The mean length per turn (worst case i.e. longest MLT possible)

(%i6) EQ6:In=2·D+2·Vt+8·Wi/4;

(EQ6) $In = 2\ Wi + 2\ Vt + 2\ D$

The core geometry coefficient

(%i7) EQ7:Kg=((Ae^2)·An·Ku)/In;

(EQ7) $$Kg = \frac{Ae^2\ An\ Ku}{In}$$

Develop the core geometry coefficient as a fuction of only the primary core dimensions by a series of substitutions.

(%i8) EQ20:subst(EQ1,EQ6);

(EQ20) $In = 2\ (W - 2\ Vt) + 2\ Vt + 2\ D$

(%i9) EQ21:subst(EQ3,EQ7);

(EQ21) $$Kg = \frac{An\ D^2\ Ku\ Vt^2}{In}$$

(%i10) EQ22:subst(EQ4,EQ21);

(EQ22) $$Kg = \frac{D^2\ Ku\ (H - 2\ Vt)\ Vt^2\ (W - 2\ Vt)}{In}$$

Core geometry corefficient in terms of Ku, D, Vt (= Ht), W and H

(%i11) EQ23:ratsimp(subst(EQ20,EQ22));

(EQ23) $$Kg=-\frac{\left(2\,D^2\,Ku\,Vt^3-D^2\,H\,Ku\,Vt^2\right)\,W-4\,D^2\,Ku\,Vt^4+2\,D^2\,H\,Ku\,Vt^3}{2\,W-2\,Vt+2\,D}$$

Differentiate with respect to Vt and set to zero to set the maxima.

(%i12) EQ24:diff(rhs(EQ23)=0,Vt);

(EQ24) $$-\frac{2\left(\left(2\,D^2\,Ku\,Vt^3-D^2\,H\,Ku\,Vt^2\right)\,W-4\,D^2\,Ku\,Vt^4+2\,D^2\,H\,Ku\,Vt^3\right)}{\left(2\,W-2\,Vt+2\,D\right)^2}-\frac{\left(6\,D^2\,Ku\,Vt^2-2\,D^2\,H\,Ku\,Vt\right)\,W-16\,D^2\,Ku\,Vt^3+6\,D^2\,H\,Ku\,Vt^2}{2\,W-2\,Vt+2\,D}=0$$

Solve for the value of Vt (=Ht) which yields the maximum core geometry coefficient.

(%i13) EQ25:solve(EQ24,Vt)[2];

(EQ25) $$Vt=\left(\frac{\sqrt{3}\,\%i}{2}+\frac{-1}{2}\right)\left(\mathrm{sqrt}\left(-252\,W^6-\left(1296\,D-568\,H\right)\,W^5-\left(511\,H^2-2740\,D\,H+2412\,D^2\right)\,W^4-\left(-179\,H^3+2338\,D\,H^2-4692\,D^2\,H+1944\,D^3\right)\,W^3-\left(17\,H^4-718\,D\,H^3+3724\,D^2\,H^2-3416\,D^3\,H+576\,D^4\right)\,W^2-\left(52\,D\,H^4-900\,D^2\,H^3+2472\,D^3\,H^2-896\,D^4\,H\right)\,W-36\,D^2\,H^4+360\,D^3\,H^3-576\,D^4\,H^2\right)/\left(8\,3^{7/2}\right)+\frac{\left(4\,D+H+5\,W\right)^3}{729}+\frac{W\,H\,D+W^2\,H}{2}-\frac{\left(4\,D+H+5\,W\right)\left(\left(6\,H+6\,W\right)\,D+7\,W\,H+6\,W^2\right)}{36}}{6}\right)^{(1/3)}-\left(\frac{-1}{2}-\frac{\sqrt{3}\,\%i}{2}\right)\left(\frac{\left(6\,H+6\,W\right)\,D+7\,W\,H+6\,W^2}{36}-\frac{\left(4\,D+H+5\,W\right)^2}{81}\right)/\left(\mathrm{sqrt}\left(-252\,W^6-\left(1296\,D-568\,H\right)\,W^5-\left(511\,H^2-2740\,D\,H+2412\,D^2\right)\,W^4-\left(-179\,H^3+2338\,D\,H^2-4692\,D^2\,H+1944\,D^3\right)\,W^3-\left(17\,H^4-718\,D\,H^3+3724\,D^2\,H^2-3416\,D^3\,H+576\,D^4\right)\,W^2-\left(52\,D\,H^4-900\,D^2\,H^3+2472\,D^3\,H^2-896\,D^4\,H\right)\,W-36\,D^2\,H^4+360\,D^3\,H^3-576\,D^4\,H^2\right)/\left(8\,3^{7/2}\right)+\frac{\left(4\,D+H+5\,W\right)^3}{729}+\frac{W\,H\,D+W^2\,H}{2}-\frac{\left(4\,D+H+5\,W\right)\left(\left(6\,H+6\,W\right)\,D+7\,W\,H+6\,W^2\right)}{36}}{6}\right)^{(1/3)}+\frac{4\,D+H+5\,W}{9}$$

Substitute values (in meters) for UU 93/76/16. The core thickness, Vt (= Ht) is 28mm for this standard catalogue core. It would be surprising if the manufacturer has not optimised the core so we may expect about 28mm to be the optimal Vt value.

(%i14) EQ26:subst([D=0.016,H=0.076·2,W=0.093],EQ25);

(EQ26) $$Vt=\left(\frac{0.001091374054643246\ \%i}{3^{7/2}}+1.189004629629619\ 10^{-5}\right)^{1/3}\left(\frac{\sqrt{3}\ \%i}{2}+\frac{-1}{2}\right)+\frac{8.819444444444439\ 10^{-4}\left(\frac{-1}{2}-\frac{\sqrt{3}\ \%i}{2}\right)}{\left(\frac{0.001091374054643246\ \%i}{3^{7/2}}+1.189004629629619\ 10^{-5}\right)^{1/3}}+0.07566666666666666$$

The expression for Vt is complex but, the complex part may be shown to vanish (is zero).

(%i15) EQ27:realpart(EQ26);

(EQ27) $$Vt=8.819444444444439\ 10^{-4}\left(-16.83640599798942\sqrt{3}\sin\left(\frac{\operatorname{atan}\left(\frac{91.78888184676067}{3^{7/2}}\right)}{3}\right)-16.83640599798942\cos\left(\frac{\operatorname{atan}\left(\frac{91.78888184676067}{3^{7/2}}\right)}{3}\right)\right)-0.01484877473433788\sqrt{3}\sin\left(\frac{\operatorname{atan}\left(\frac{91.78888184676067}{3^{7/2}}\right)}{3}\right)-0.01484877473433788\cos\left(\frac{\operatorname{atan}\left(\frac{91.78888184676067}{3^{7/2}}\right)}{3}\right)+0.07566666666666666$$

The optimum Vt value (in meters)

(%i16) float(%);

(%o16) $Vt=0.02950763905335568$

The imaginary part is indeed zero.

(%i17) EQ28:imagpart(EQ26);

(EQ28)
$$0 = 8.819444444444439\ 10^{-4}\left(16.83640599798942\ \sin\left(\frac{\operatorname{atan}\left(\frac{91.78888184676067}{3^{7/2}}\right)}{3}\right) - 16.83640599798942\ \sqrt{3}\ \cos\left(\frac{\operatorname{atan}\left(\frac{91.78888184676067}{3^{7/2}}\right)}{3}\right)\right) - 0.01484877473433788\ \sin\left(\frac{\operatorname{atan}\left(\frac{91.78888184676067}{3^{7/2}}\right)}{3}\right) + 0.01484877473433788\ \sqrt{3}\ \cos\left(\frac{\operatorname{atan}\left(\frac{91.78888184676067}{3^{7/2}}\right)}{3}\right)$$

Figure 2: Numerical calculation to demonstrate the shape of the curve of Kg as a function of core thickness, Vt (= Ht). ~ 28mm is demonstrably optimal by numerical methods as well.

